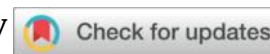




Optimization of Mechanical and Hydraulic Drilling Parameters for Maximizing Rate of Penetration Using Response Surface Methodology



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Abstract

Optimizing the Rate of Penetration (ROP) is a critical factor for enhancing productivity and reducing costs in well drilling operations. This study aims to identify the optimal operating conditions for achieving a maximum ROP by systematically investigating the influence of key drilling parameters. Response Surface Methodology (RSM) was employed as a powerful tool for process modelling and optimization, examining the individual and interactive effects of three crucial operational variables: Weight on Bit (WOB), Rotation Speed (RPM), and Mud Flow Rate (Q).

Experimental data were collected and analysed to develop a precise statistical mathematical model describing the relationship between these parameters and the ROP. Analysis of the results demonstrated that the developed model possesses high predictive capability and significant statistical accuracy, making it suitable for determining the optimal combination of parameters for a rational and efficient operational performance. This study highlights the practical importance of statistical optimization techniques, such as RSM, in drilling engineering to enhance productivity while ensuring operational efficiency.

Keywords: *Drilling Optimization, Response Surface Methodology (RSM), Rate of Penetration (ROP) Modelling, Weight on Bit (WOB), Rotation Speed (RPM), Mud Flow Rate, Experimental Design.*

1. Introduction

Oil is considered a fundamental element in the global economic dynamics, significantly impacting various sectors worldwide. To increase production, enhancing the performance of the drilling process is essential. Increasing the Rate of Penetration (ROP) plays a critical role in improving oil-drilling efficiency. A higher ROP allows for faster drilling, thus reducing costs, optimizing resources, and minimizing risks, while also enhancing the overall productivity of the operation. Operators employ various strategies such as optimizing drilling parameters, using more powerful drill rigs, and improving drill bits to maximize ROP while maintaining high safety standards. This research primarily focuses on analysing the impact of drilling parameters, such as Weight on Bit (WOB), Rotation Speed (RPM), and Flow Rate (Q), on the rate of penetration. Drilling parameters are factors

that affect the ROP, which are categorised into two main groups: mechanical parameters related to the type and shape of the tool, weight, and rotation speed, and hydraulic parameters such as flow rate, pressure, and the characteristics of the drilling fluid. Understanding the relationships between these parameters enables the development of both theoretical and practical concepts for better controlling drilling operations. In this context, the current research aims to clarify the empirical relationships between various factors influencing the rate of penetration using the Response Surface Method (RSM), specifically Box-Behnken experimental designs. Box-Behnken designs were chosen for their efficiency, statistical robustness, and ease of interpretation when exploring and optimizing processes or systems[1].

Since the global recognition of the critical demand for hydrocarbons in both utilization and substantial financial investment, oil exploration and exploitation have become central factors in driving technological advancements and profit expansion. Moreover, it is widely acknowledged that the oil and gas sector is increasingly focusing on optimizing drilling process designs to lower operational expenses while enhancing operational efficiency [2]. Rotary blasting hole drills are extensively employed worldwide in surface mineral extraction for waste removal purposes. The precise estimation of the penetration rate for rotary drill rigs is highly significant within the context of rock drilling, particularly in the fields of geology and petroleum technology [3.4]. Accurately estimating the penetration rate is essential in the process of mine construction. The assessment of total drilling expenses can be achieved through the use of predictive formulas [5]. Additionally, predictive formulas can be employed to identify the most suitable type of drilling rig for specific situations. Rotary tricone bits, including tungsten carbide inserts, are widely favored as the primary drilling tools for deep holes with substantial diameters in extensive surface mining processes[6]. Over time, exploration rates have increased due to the adoption of more powerful drills and enhanced management of operational factors. This, in turn, has led to higher mining output and reduced drilling costs.

Today, deep drilling practices hold significant importance and are widely promoted within the oil and gas industry. However, this technique is not without its challenges, primarily due to the substantial depth involved and the complex process of tool replacement, compounded by anomalies encountered within formation layers. These factors often lead to inconsistent results, causing mechanical issues that ultimately reduce the tool's penetration depth. In this context, there is a shared interest among industry experts and academics in designing and developing novel drilling techniques to improve drilling operation performance [7.8]. Enhancing drilling operation efficiency and achieving superior performance levels require the optimization of various drilling parameters, including the weight of the drill bit, the rotational speed of the drilling apparatus, the rock's resistance, and the properties of the drilling mud. This optimization primarily revolves around achieving the highest drilling rate while minimizing costs and the mass of the rock drillable indicator [9.10]. Much attention has been given to improving the quality of the drilling process. Garnier and Van Lingen [11] focused on specific phenomena that could affect drilling operations. Response Surface Methodology (RSM) is one of the most effective approaches for understanding and modelling such phenomena. RSM aims to systematically and efficiently explore the correlation between input factors and response variables in order to optimize procedures, products, or systems while minimizing the need for extensive experimentation and resources [12]. RSM is regarded as a crucial component of experimental design for developing new processes and improving their performance. This methodology was also developed to enhance products and systems, with the goal of optimizing the load component and reducing process response instability [13]. In general, RSM consists of a collection of statistical and mathematical techniques that are highly effective in analysing and addressing problems where

multiple factors influence the response variable. Its goal is to improve this response[14,15]. The objective of RSM is to determine the optimal empirical design with the fewest possible design repetitions. Its use in empirical design dates back to the late 1990s [16]. This technique has been used by numerous researchers, such as Panagiotis et Angelos [17]. To investigate how the process parameters of fiber laser percussion drilling influence the geometric characteristics of 1.0 mm thick Inconel 718, experiments were conducted using RSM by Moradi and Mohazabpak [18]. The primary aim of this study is to develop mathematical simulations to predict the propulsion force and cutting torque in the context of drilling operations. Salehnezhad et al. [19] utilized RSM to optimize and improve the properties of drilling mud. By using the box-Behnken design within the RSM framework, Zhang [20] conducted several laser drilling experiments. The goal of these experiments was to determine the specific energy of rock by varying three key empirical factors: laser power, irradiative time, and spot diameter. Alakbari et al. [21] introduced new statistical empirical correlations for prediction through the application of RSM. RSM was used to establish mathematical relationships between factors and responses, as well as to clarify the interactions among variables. Surekha et al. [22] attempted to examine the effect of aluminum powder on the electrical discharge machining (EDM) of EN-19 alloy steel. Using surface response modeling, a relationship was established between the responses and the operational factors of the procedure.

In recent years, advanced methods have been developed to optimize drilling parameters, focusing on increasing the Rate of Penetration (ROP) and reducing operational costs. A recent study demonstrated the use of machine learning algorithms to analyse field data and optimize drilling parameters such as Weight on Bit (WOB), Rotation Speed (RPM), and Flow Rate (Q), resulting in a reduction of prediction error for the ROP from 18.72% to 10.56%. [23]

Additionally, Response Surface Methodology (RSM) with Box-Behnken design was applied to optimize drilling mud properties, helping to improve fluid stability and reduce fluid loss during drilling. [24]

These studies highlight the importance of integrating modern techniques, such as machine learning and advanced experimental design, to improve drilling operations and enhance their efficiency.

2. Material and Methods

2.1. Materials Used in the Study

The experimental tests conducted in this study were performed using the Simulators Company, a petroleum-drilling simulator from the National Algerian Drilling Company (ENAFOR). This simulator is specifically designed to replicate the structure and functionality of a conventional drilling rig. It includes traditional drilling controls, analog instruments, and a manual brake system, all integrated with 3D graphical representations. This configuration provides a realistic simulation environment that allows for the testing of various drilling conditions without the need for actual field operations. The Simulators Company offers a versatile platform for conducting different experiments, enabling the examination of drilling parameters and the optimization of drilling processes in a controlled setting.



Figure 1. Drilling Simulator.

Table 1. Shows the values of the drilling parameters used

Table 1. Values of the parameters.

Factors	Symbol	Unit	Levels		
Weight on the Bit	WOB	MT	5	10	15
Rotational Speed	RPM	rpm	60	80	100
Mud Flow Rate	Q	l/min	989	1320.5	1652

The results of the penetration rate from the experiments, conducted according to the Box-Behnken design, are presented in Table 2.

Table 2. Experimental Penetration Rate(ROP).

	Factor 1	Factor 2	Factor 3	Response
N°	A: WOB Weight on Bit (MT)	B: RPM Rotational Speed (rpm/min)	C: Q Mud Flow Rate (l/min)	ROP The experimental penetration rate (m/h)
1	5	80	1652	1.5
2	10	100	1652	3.3
3	15	60	1320.5	4.4
4	15	80	989	5.4
5	5	60	1320.5	1.2
6	5	100	1320.5	1.5
7	10	60	1652	2.6
8	10	100	989	4.3
9	10	80	1320.5	3.4
10	10	60	989	2.8
11	10	80	1320.5	3.4
12	5	80	989	1.6
13	15	100	1320.5	6.4
14	15	80	1652	5.3
15	10	80	1320.5	3.4

16	10	80	1320.5	3.4
17	10	80	1320.5	3.4

To explain and identify the relationship between the different factors and the response ROP, we use the Response Surface Methodology (RSM).

2.2. Response Surface Methodology (RSM) and Box–Behnken Design

Response Surface Methodology (RSM) is a statistical tool used for optimizing processes involving multiple variables by modeling the relationship between input factors and the response variable. The key objective of RSM is to explore the optimal levels of input variables that result in the best outcome for a system. It employs a series of designed experiments to establish mathematical models for the response variable, often using quadratic polynomials. A typical RSM model can be represented as follows:

$$Y = \beta_0 + \sum_{i=1}^k \beta_i X_i + \sum_{i=1}^k \beta_{ii} X_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^k \beta_{ij} X_i X_j + \epsilon$$

Where:

- Y : The response variable, such as extraction yield or product size.
- X_i and X_j : the input variables (factors), , these typically have three levels: -1 , 0 , and $+1$.
- β_0 : The constant or intercept term in the model.
- β_i : The linear coefficients that represent the effect of each independent variable on the response.
- β_{ii} : The quadratic coefficients that account for the curvature of the response surface.
- β_{ij} : The interaction coefficients that represent the combined effect of two independent variables (between factors i and j).on the response.
- ϵ : The model's error term.
- k : The number of independent variables or factors in the design.

Among the various experimental designs used in RSM, the **Box–Behnken Design (BBD)** is one of the most popular due to its efficiency. The BBD is a three-level design that requires fewer experimental runs compared to a full factorial design and is especially useful when interactions between variables are important. The design matrix consists of points placed at the midpoints of the edges of a cube (without axial points), where each factor is evaluated at three levels: low (-1), middle (0), and high ($+1$).

The quadratic model for the Box–Behnken design can be expressed as:

$$Y = \beta_0 + \sum_{i=1}^k \beta_i X_i + \sum_{i=1}^k \beta_{ii} X_i^2 + \sum_{i < j}^k \beta_{ij} X_i X_j$$

Where X_i are the factor levels and Y is the response. For Box–Behnken, the design matrix does not include axial points and is often represented as:

$$\text{Design Matrix} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Where factors (such as temperature, pressure, etc.) are manipulated according to the design matrix to study their effects on the response. The efficiency of the Box–Behnken design in reducing the number of experimental runs while providing comprehensive information on the interactions between variables makes it a powerful tool for optimizing complex processes. By fitting a second-order (quadratic) model to the experimental data, it helps in understanding the response surface and identifying the optimal combination of factors for the desired outcome.

Recent studies have demonstrated the versatility of the BBD in process optimization. For example, Perveen et al. (2024) employed BBD to optimize the synthesis conditions of Schiff bases and dihydropyrimidinones, achieving higher yields under optimal conditions [25]. Similarly, Shao et al. (2024) applied BBD to optimize the formulation of activated lithium slag composite cement, improving its mechanical properties [26]

3. Data Analysis and Processing

Analysis of Variance (ANOVA) is a statistical technique used to compare the means of different groups. It is used to test whether there are statistically significant differences between the means of more than two populations. If the variance is significant, it suggests that the explanatory variable (parameters) has a significant effect on the dependent variable (response).

The results of the analysis are summarised in Table 3.

Table 3. Analysis of Variance for ROP

Source	Sum of Squares	df	Mean Square	p-valeur
Model	34.56	9	3.84	< 0.0001
A-WOB	30.81	1	30.81	< 0.0001
B-RPM	2.53	1	2.53	< 0.0001

C-Q	0.2450	1	0.2450	0.0131
AB	0.7225	1	0.7225	0.0008
AC	0.0000	1	0.0000	1.0000
BC	0.1600	1	0.1600	0.0322
A ²	0.0322	1	0.0322	0.2703
B ²	0.0533	1	0.0533	0.1677
C ²	0.0059	1	0.0059	0.6237
Residual	0.1575	7	0.0225	
Total	34.72	16		

The results presented in the table indicate that the model is well-fitted. This is evident from the significantly lower sum of squared residuals (0.1575) compared to the total sum of squares due to regression (34.72). Therefore, the influence of parameters not included in the model on the ROP behavior is minimal relative to the effect of the model parameters.

The p-value of 0.0001 for the model confirms its statistical significance. Parameters with a p-value below 0.05 are considered statistically significant. In this case, the significant terms of the model include A, B, C, AB, and BC. This outcome demonstrates that the experiments yielded reliable results, indicating that the model is well-adjusted and appropriately reflects the underlying data.

4. Goodness of Fit Statistics

Table 4 presents the statistical indices used to evaluate the quality of the fit of the developed mathematical model.

Table 4. Statistical Indices

R²	0.9955
Adjusted R²	0.9896
Predicted R²	0.9274
Adequate Precision	43.8960

The predicted R² value of 0.9274 closely aligns with the adjusted R² of 0.9896, indicating that the difference between them is less than 0.2, which suggests a good model fit. Adequate Precision is a measure of the signal-to-noise ratio, with a ratio greater than 4 being desirable. The value of 43.896 indicates an adequate signal quality, which implies that the model is suitable for use in the design space.

The results of the analysis of variance and the evaluation of statistical indices demonstrate that the model is well-fitted, making it appropriate for accurately predicting the response (ROP).

5. Mathematical Modeling of ROP

The mathematical model provides a method to calculate the rate of penetration (ROP) for any given values of the three parameters within the scope of the study. The coded equation is useful for identifying the relative impact of the factors by comparing their coefficients, where high factor levels are coded as +1, and low levels are coded as -1. The quadratic response equation for ROP, calculated using DESIGN EXPERT 11 software, is expressed as:

$$\text{ROP} = 3.4 + 1.9625 \times A + 0.5625 \times B - 0.175 \times C + 0.425 \times AB - 1.24058e^{-17} \times AC - 0.2 \times BC + 0.0875 \times A^2 - 0.1125 \times B^2 - 0.0375 \times C^2.$$

Additionally, the model in real factors, which can be used to predict the response for different levels of each factor, requires the levels to be specified in the original units of the factors. However, this equation should not be used to determine the relative impact of each factor, as the coefficients are scaled to account for the units, and the intercept is not centered within the design space. The final model for ROP in real factors is:

$$\text{ROP} = -3.90966 - 0.0175 \times \text{WOB} + 0.0704591 \times \text{RPM} + 0.00278659 \times Q + 0.00425 \times \text{WOB} \times \text{RPM} - 6.84785e^{-20} \times \text{WOB} \times Q - 3.01659e^{-0.5} \times \text{RPM} \times Q + 0.0035 \times \text{WOB}^2 - 0.00028125 \times \text{RPM}^2 - 3.41243e^{-0.7} \times Q^2$$

6. Influence of Different Factors on ROP

The following study investigates the influence of various factors, including weight on the bit (WOB), rotation speed (RPM), and flow rate (Q), on the rate of penetration (ROP), as shown in Figures 2, 3, and 4.

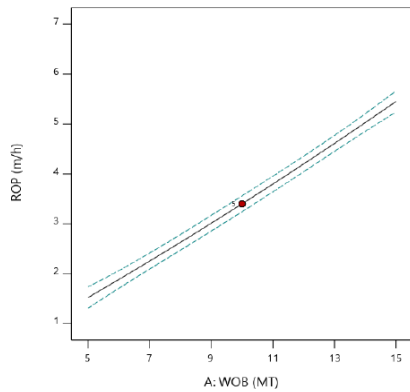


Figure 2. Variation of ROP as a function of WOB.

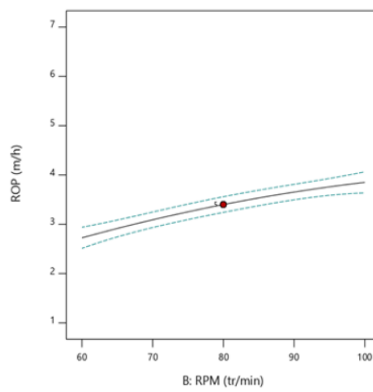


Figure 3. Variation of ROP as a function of RPM.

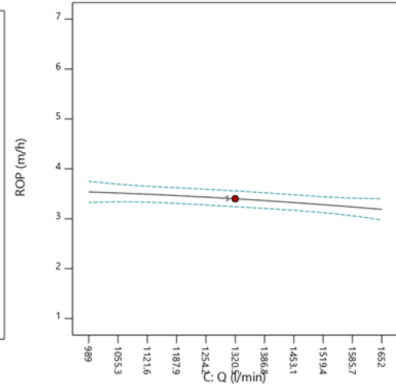


Figure 4. Variation of ROP as a function of Flow Rate (Q).

In Figure 2, a proportional relationship exists between ROP and WOB, where an increase in the weight applied to the tool leads to a higher ROP. Figure 3 demonstrates the effect of RPM on ROP, illustrating that an increase in rotation speed results in a moderate rise in ROP, attributed to the shorter

interval between cutting element impacts on the rock. Additionally, Figure 4 shows that ROP remains relatively stable with an increase in flow rate (Q), but beyond a specific flow rate of 1320.5 liters per minute, ROP starts to decrease, indicating a diminishing return from further increases in flow rate.

7. Interaction of Factors

The variation in the rate of penetration (ROP) as a function of the analysed parameters can be visualised through a three-dimensional (3D) surface response graph. This representation demonstrates the fluctuation of ROP in relation to the interaction between two parameters, while keeping the third parameter constant.

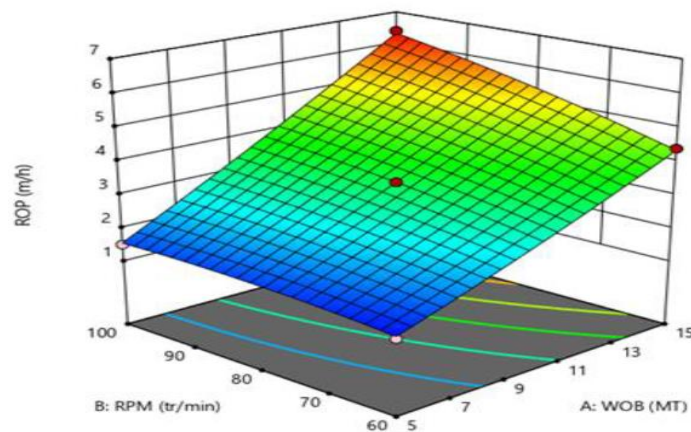


Figure 5. Variation of ROP as a function of WOB and RPM.

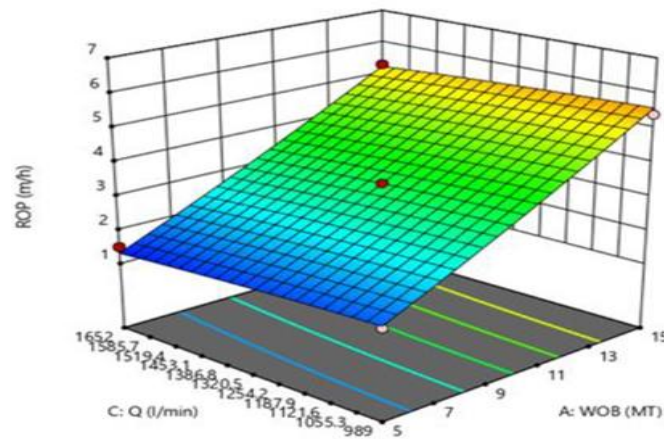


Figure 6. Variation of ROP as a function of WOB and Q.

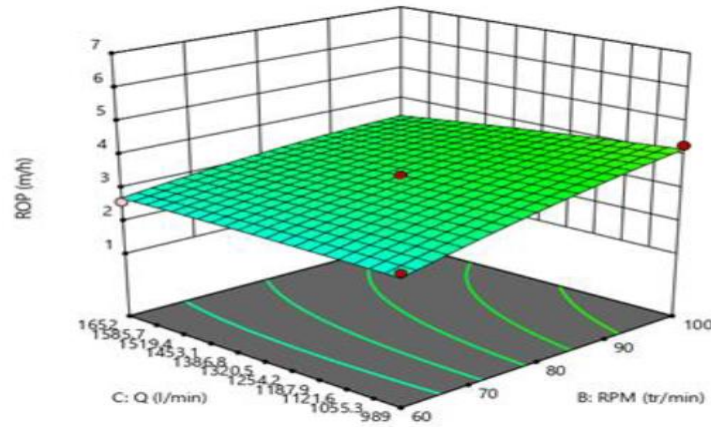


Figure 7. Variation of ROP as a function of RPM and Q.

The graphs in Figures 5, 6, and 7 demonstrate the effects of various factors on the rate of penetration (ROP). In Figure 5, it is observed that increasing both the weight on the tool (WOB) and the rotation speed (RPM) significantly enhances the ROP. Similarly, Figure 6 shows that when the weight on the tool (WOB) and the flow rate (Q) are increased, there is a notable improvement in the ROP. Lastly, Figure 7 illustrates that the ROP remains stable initially, followed by a slight increase as both the rotation speed (RPM) and flow rate (Q) increase

8.Validation of Results

Regression is a widely recognized and commonly used statistical technique to establish a relationship between a dependent variable and one or more independent variables. When examining the relationship between a dependent variable and multiple independent variables, the application of regression requires a causal relationship between the variables included in the model. The regression line, which illustrates the predicted rate of penetration based on the observed (real) rate of penetration values, is shown in Figure 8.

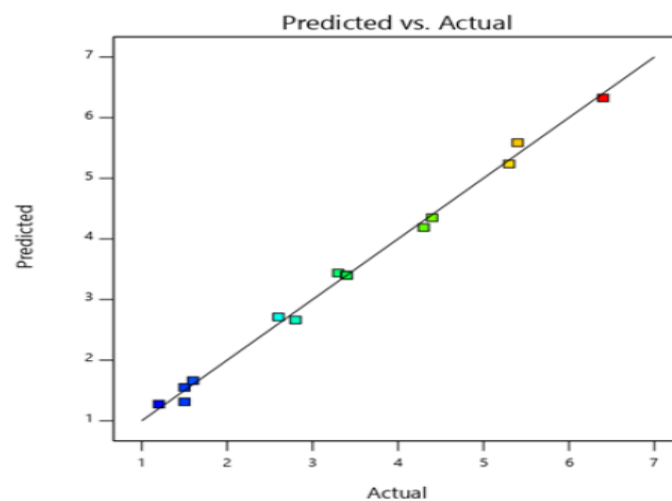


Figure 8. Predicted ROP as a function of measured ROP.

The graph demonstrates that the scatter plot representing the measured response values is very close to the regression line, indicating a strong convergence between the two. This result confirms that the model has good descriptive quality and is highly useful for predicting the response behavior.

9. Optimization of Rate of Penetration (ROP)

The primary goal of this study is to maximize the rate of penetration (ROP) by analysing the variations in several key factors. To determine the optimal value of a multivariable function, it is essential to find the points where the partial derivatives with respect to each factor namely, weight on the bit (WOB), rotation speed (RPM), and mudflow rate (Q) equal zero. Table 5 displays the results obtained from solving the optimization equations using Design Expert 11 software. The optimal values for the parameters are as follows:

Table 5. Optimal values for the parameters.

Weight on the tool (WOB)	Rotation speed (RPM)	Mud flow rate (Q)	Rate of penetration (ROP)
14.338 TM	91.080 rpm	1456.799 l/min	5.526 m/h

Conclusion

This study aimed to optimize the rate of penetration (ROP) by adjusting various parameters, such as weight on the bit (WOB), rotation speed (RPM), and mud flow rate (Q). To achieve this, we employed the Response Surface Methodology (RSM). The results of this study clearly demonstrate that the parameters examined WOB, RPM, and Q significantly influence the rate of penetration. Among these, the effect of WOB was found to be the most significant, while the combined effect of WOB and RPM had the greatest impact on ROP compared to other parameter combinations. The study confirms that the highest ROP (5.526 m/h) was achieved with the following optimal values: WOB = 14.338 TM, RPM = 91.080 rpm and Q = 1456.799 l/min.

The application of RSM validates the effectiveness and accuracy of the developed model, as evidenced by the strong correlation between the predicted and experimental data. This method allows for the precise prediction of ROP. Furthermore, incorporating additional parameters, such as rock type or tool geometry, can provide even more precise results in understanding the behavior of ROP and improving drilling performance.

References

[1]. Golem, A. Golem: An algorithm for robust experiment and process optimization. arXiv. (2021).

- [2]. Tuna, E. (2010). Real-time-optimization of drilling parameters during drilling operations (Thesis). Middle East Technical University.
- [3]. Saeidi, O., Torabi, S. R., Ataei, M., and Rostami, J. (2014). A stochastic penetration rate model for rotary drilling in surface mines. *International Journal of Rock Mechanics and Mining Sciences* 68: 55-65.
- [4]. Saeidi, O., Rostami, J., Ataei, M., and Torabi, S. R. (2014). Use of digital image processing techniques for evaluating wear of cemented carbide bits in rotary drilling. *Automation in Construction* 44: 140-151.
- [5]. Shad, H. I. A., Sereshki, F., Ataei, M., and Karamoozian, M. (2018). Prediction of rotary drilling penetration rate in iron ore oxides using rock engineering system. *International Journal of Mining Science and Technology* 28(3): 407-413.
- [6]. Piri, M., Mikaeil, R., Hashemolhosseini, H., Baghbanan, A., and M. Ataei, (2021). Study of the effect of drill bits hardness, drilling machine operating parameters, and rock mechanical parameters on noise level in hard rock drilling process. *Measurement* 167: 108447.
- [7]. Doiron, H.H. and Deane, J.D. (1982). Effects of hydraulic parameter cleaning variations on rate of penetration of soft formation insert bits. *SPE Annual Technical Conference and Exhibition*. Society of Petroleum Engineers, ISBN: 978-1-55563-666-1.
- [8]. Harold, H. and John, D. (1982). Effect of hydraulic parameter cleaning variations on rate of penetration of soft formation insert bit. Paper SPE 11058 Presented at the 57th Annual Fall Technical Conference and Exhibition, New Orleans, CA.
- [9]. Ataei, M., KaKaie, R., Ghavidel, M., and Saeidi, O. (2015). Drilling rate prediction of an open-pit mine using the rock mass drillability index. *International Journal of Rock Mechanics and Mining Sciences* 73: 130-138.
- [10]. Irawan, S., Rahman, A., and Tunio, S. (2012). Optimization of weight on bit during drilling operation based on rate of penetration model. *Research Journal of Applied Sciences, Engineering and Technology* 4(12): 1690-1695.
- [11]. Garnier, A. and Van Lingen, N. (1959). Phenomena affecting drilling rates at depth.
- [12]. Alipenhani, B., Majdi, A., and Amnieh, H.B. (2022). Capability assessment of rock mass in block caving mining method based on numerical simulation and response surface methodology. *Journal of Mining and Environment* 13(2): 579-606.
- [13]. Taguchi, G. (1987). System of experimental design; engineering methods to optimize quality and minimize costs.
- [14]. Gunst, R.F. (1996). Response surface methodology: process and product optimization using designed experiments. Taylor & Francis.
- [15]. Millheim, K.K. (1983). An engineering simulator for drilling: part II, *SPE Annual Technical Conference and Exhibition*. Soc. Pet. Eng. ISBN: 978-1-55563- 652-4.
- [16]. Montgomery, D.C. and Raymond, H.M. (2002). Response Surface Methodology: Process and Product Optimization using Designed Experiments. John Wiley, New York.
- [17]. Kyratsis, P., Markopoulos, A.P., Efkolidis, N., Maliagkas, V., and Kakoulis, K. (2018). Prediction of thrust force and cutting torque in drilling based on the response surface methodology. *Machines* 6(2): 24.
- [18]. Moradi, M. and Mohazabpak, A. R. (2018). Statistical Modelling and Optimization of Laser Percussion Microdrilling of Inconel 718 Sheet using Response Surface Methodology (RSM). *Lasers in Engineering* (Old City Publishing), 39.

- [19]. Salehnezhad, L., Heydari, A., and Fattahi, M. (2019). Experimental investigation and rheological behaviors of water-based drilling mud contained starch-ZnO nanofluids through response surface methodology. *Journal of Molecular Liquids* 276: 417- 430.
- [20]. Zhang, W., Huang, Z., Kang, M., Shi, M., Deng, R., Yan, Y., and Zhu, Q. (2021). Research on multivariate nonlinear regression model of specific energy of rock with laser drilling based on response surface methodology. *Optics Communications* 489:126865.
- [21]. Alakbari, F.S., Mohyaldinn, M.E., Ayoub, M.A., Muhsan, A.S., and Hassan, A. (2021). Apparent and plastic viscosities prediction of water-based drilling fluid using response surface methodology. *Colloids and Surfaces A: Physicochemical and Engineering Aspects* 616: 126278.
- [22]. Surekha, B., Sree Lakshmi, T., Jena, H., and Samal, P. (2021). Response surface modelling and application of fuzzy grey relational analysis to optimise the multi response characteristics of EN-19 machined
- [23] Raed H. Allawi ^{ad}, Watheq J. Al-Mudhafar^b, Mohammed A. Abbas ^b, David A. Wood Leveraging boosting machine learning for drilling rate of penetration (ROP) prediction based on drilling and petrophysical parameters. *Artificial Intelligence in Geosciences* Volume 6, Issue 1, June 2025, 100121
- [24] Joy Ehimwenma Ossai*, Eghe Amenze Oyedoh, Vera OgheneovoMagbuwe, Messiah Innocent Atapia-Optimization of the Formulation of Drilling Mud using Box – Behnken Design. *Journal of Energy Technology and Environment* Vol. 7(2) 2025 pp. 166-173 ISSN-2682-583x166
- [25] Perveen, R., et al. (2024). Optimization study and application of Box–Behnken model in the synthesis of Schiff bases and dihydropyrimidinones. *Scientific Reports*, 13(1), 5637.
- [26] Shao, W., et al. (2024). Experimental study based on Box–Behnken design and response surface methodology for optimization proportioning of activated lithium slag composite cement-based cementitious materials. *Materials*, 17(11), 2651.